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Abstract
We systemically study the correlation properties of two-dimensional electrons in the lowest as well as in higher Landau levels by the exact diagonalization method. The Laughlin liquid state and the Wigner crystal are shown in the lowest Landau levels. In higher Landau levels, bubbles and stripes are found. The results coincide well with the current understanding of the strongly correlated electrons in quantum Hall regimes.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In the early 1930s, Wigner suggested that two-dimensional (2D) electrons may crystallize into a triangular lattice in the low-density and low-temperature limit where the electron–electron interactions dominate over the kinetic energy. In an ideally clean 2D system, the critical r_s ($r_s = U/\epsilon_F$, corresponding to the ratio of the Coulomb energy scale U to the kinetic energy scale of the Fermi energy ϵ_F) was suggested to be 37 ± 5 from quantum Monte Carlo simulations [1]. A strong magnetic field perpendicular to the 2D plane can effectively localize electron wavefunctions while keeping the kinetic energy controlled through Landau orbital quantization [2]. Since this lessens the otherwise severe low-density condition, it is believed that the Wigner crystal (WC) can be stabilized in a sufficiently strong magnetic field [3–5]. Approximate calculations [6] have shown that the WC becomes the lowest energy state when $\nu < 1/6$ for the GaAs/AlGaAs electron system and around $\nu = 1/3$ for the hole system, where ν is the filling factor of the Landau level (LL) defined by $\nu = 2\pi l^2 \rho_e$, with ρ_e being the density of 2D electrons and $l = (\hbar c/eB)^{1/2}$ the magnetic length.

On the other hand, the fractional quantum Hall effect (FQHE) [7, 8] was first discovered at the lowest LL in the 2D electron systems (2DES). This remarkable phenomenon occurring at certain unique values of odd-denominator filling factors $\nu = 1/3, 1/5, \ldots$ has been associated

with the formation of the uniform incompressible quantum state, or the Laughlin liquid. The traditional alternative to the Laughlin liquid is a charge density wave (CDW), which does not exhibit the FQHE. Hall plateaus with other odd-denominator fillings were also found in the lowest LL as well as in higher LLs. Furthermore, there have appeared a bundle of enigmatic phenomena for half-filled LLs since the discovery of the Hall metallic state at $\nu = 1/2$ and the only observed Hall plateau at the even-denominator filling factor ($\nu = 5/2$) in the monolayer system. Halperin *et al* suggested that the $\nu = 1/2$ state may be viewed as a spin-polarized Fermi liquid of composite fermions (CFs) [9, 10]. The $\nu = 5/2$ quantum Hall state is now widely accepted as the pairing of CFs in a vanishing effective magnetic field [11–13]. Works on multilayer Hall systems have also been done intensively and extensively [14–18]. Because of interlayer coupling, Hall plateaus at filling fractions not seen in the monolayer system such as at $\nu = 1/2$ can be distinguished clearly in bilayer systems [19].

Recently, a series of magneto-transport experiments on high-mobility samples in the GaAs/Al_xGa$_{1-x}$As heterostructures revealed new classes of correlated many-electron states in higher LLs. The most prominent findings are the giant anisotropy in the resistivity near the half-filling of the topmost LL [20, 21] and the re-entrant integral quantum Hall (RIQH) states in the flanks of these same levels. It is believed that the highly anisotropic transport is related to the formation of the unidirectional CDW state, or the stripe phase, which was theoretically predicted by Koulakov *et al* based on the Hartree–Fock treatment of the high LLs [22]. Eisenstein *et al* revealed that the anisotropy occurs when the 2D system is applied by an in-plane magnetic field [23]. The easy direction of transport is perpendicular to the in-plane field. Specifically, no anisotropy is observed for the $\nu = 5/2$ state when the direction of the magnetic field is perpendicular to the sample plane, whereas distinct anisotropy occurs when an extra in-plane field is applied. Recent theoretical study shows that this system may be classified by their symmetries, which are highly analogous to those of liquid crystals. The RIQH effect was thought to be the depinning and sliding of the WC and reformation of the bubble phase.

There are many works that directly diagonalize the Hamiltonian for few-electron systems [24–26]. Calculations for systems involving the electron spins are also attempted but these have not been so successful because of the limitation of the dimensions of Hilbert space [27]. Recently, Shibata and Yoshioka have successfully applied the density matrix renormalization group method (DMRG) to the quantum Hall problem [28]. In this work, we will systemically investigate the correlated properties of 2D electron systems in the lowest LL as well as in higher LLs. We find that apart from the usual liquid and WC states in the lowest LL, bubble states and stripe states form in higher LLs. These states are called CDW. Our results coincide with the prevailing explanation of the anisotropic transportation measurements.

2. Method

We consider N_e electrons moving in a rectangular plane of size $a \times b$ with a high magnetic field B pointing in the z-direction. The number of states in each degenerate LL is $N_s = ab/2\pi l^2$. In the Landau gauge $\vec{A} = (0, Bx)$, a basis of the nth LL consists of N_s degenerate states which is labelled by an integer j with $1 \leqslant j \leqslant N_s$ is

$$\tilde{\psi}_{n,j} = N_n e^{iX_j y/l^2} H_n((x - X_j)/l) e^{-(x-X_j)^2/2l^2}, \tag{1}$$

where $X_j = 2\pi l^2 j/b$. $H_n(x)$ is the Hermitian polynomial with n the LL index and $N_n = (2^n n! \sqrt{\pi} l b)^{-1/2}$ is the normalization constant. The wavefunction $\tilde{\psi}_{n,j}$ is periodic in the y-direction. To make the wavefunction periodic in the x-direction as well, the following

combination of $\tilde{\psi}_{n,j}$ should be used:

$$\psi_{n,j}(\vec{r}) = \left[\frac{1}{b\sqrt{\pi}l}\right]^{1/2} \sum_{k=-\infty}^{\infty} H_n(x - X_j) e^{[i(X_j+ka)y/l^2 - (X_j+ka-x)^2/2l^2]}. \tag{2}$$

The electrons in the cell interact with each other and with the uniform positive background charge by Coulomb interaction. Because of the periodic boundary condition, the Coulomb interaction potential in real space is given by

$$V(\vec{r}_1 - \vec{r}_2) = \sum_s \sum_t \frac{e^2}{\epsilon|\vec{r}_1 - \vec{r}_2 + sa\hat{x} + tb\hat{y}|}$$
$$= \frac{1}{ab} \sum_{\vec{q}} \frac{2\pi e^2}{\epsilon|\vec{q}|} e^{-i\vec{q}\cdot(\vec{r}_1-\vec{r}_2)}, \tag{3}$$

where $q_x = 2\pi s/a$, $q_y = 2\pi t/b$ with s, t integers. The Hamiltonian is expressed in the second quantized picture as

$$\hat{H} = \sum_j \epsilon_M a_j^\dagger a_j + \sum_{\{j\}} A_{j_1 j_2 j_3 j_4} a_{j_1}^\dagger a_{j_2}^\dagger a_{j_3} a_{j_4}, \tag{4}$$

where $a_j^\dagger$ (a_j) is the creation (annihilation) operator of the state $\psi_{n,j}$. ϵ_M is the classical Coulomb energy of the WC with a rectangular unit cell, which is given by [24]

$$\epsilon_M = -\frac{e^2}{\epsilon\sqrt{ab}} \left[2 - \sideset{}{'}\sum_{k_1,k_2} \phi_{-1/2}[\pi(\lambda k_1^2 \lambda^{-1} k_2^2)]\right], \tag{5}$$

where $\lambda = a/b$, and the sum over k_1 and k_2 excludes $k_1 = k_2 = 0$, and $\phi_\alpha(z) = \int_1^\infty t^\alpha e^{-zt} dt$. By constraining ourselves to the topmost LL n, the matrix element A is given by

$$A_{j_1 j_2 j_3 j_4} = \frac{1}{2} \int d^2\vec{r}_1 \int d^2\vec{r}_2 \psi_{n,j_1}^*(\vec{r}_1)\psi_{n,j_2}^*(\vec{r}_2) V(\vec{r}_1 - \vec{r}_2) \psi_{n,j_3}(\vec{r}_2)\psi_{n,j_4}(\vec{r}_1)$$
$$= \delta'_{j_1+j_2,j_3+j_4} \frac{1}{2ab} \sum_{\vec{q}} \delta'_{j_1-j_4,q_y b/2\pi} \frac{2\pi e^2}{\epsilon|\vec{q}|} e^{-\frac{1}{2}\vec{q}^2 l^2 - iq_x(X_{j_1}-X_{j_3})} \left[L_n\left(\frac{\vec{q}^2 l^2}{2}\right)\right]^2, \tag{6}$$

where the Kronecker δ'_{j_1,j_2} indicates $j_1 = j_2 \pmod{N_s}$ and the $L_n(x)$ are Laguerre polynomials. In the following discussions, we only consider the correlation between electrons in the topmost LL since the lower LLs are completely filled and effects from electrons in lower LLs can be viewed as a uniform background. In the mean-field approximation, this background plays the role of screening the Coulomb interactions between electrons in the topmost LL. But this screening effect is not critical, and in our computation we can safely neglect it [29].

For a system of N_e polarized electrons with N_s degenerate states, the number of electrons at the topmost LL (the nth LL) is $N_n = N_e - 2n \times N_s$. The lower LLs are fully filled and the filling factor at the topmost LL is then $\nu_n = N_n/N_s$. We numerically diagonalize the Hamiltonian by taking account of several symmetries of the system. First, one notes that the dimension of the Fock space is $C_{N_s}^{N_n}$. Owing to the translational symmetry along the y-axis, $J_{tot} \equiv \sum_{i=1}^{N_s} j_i \pmod{N_s}$. This symmetry will reduce the dimension of the Fock space by nearly N_s times. The translation symmetry along the x-direction implies that two systems with values of J_{tot} which differ by a multiple of N_n are equivalent to that with J_{tot}, which means only a small number of J_{tot} that leads to different results.

In figure 1 we show the low-lying energy spectrum versus J_{tot} in the lowest LL for nine electrons at $\nu = 1/2$ and six electrons at $\nu = 1/3$, respectively. The energy is minimized for the optimal aspect ratio of the sample $a/b = \sqrt{3}$ and is measured in units of e^2/l. The shorter dashes indicate degenerate states with the same J_{tot}. Our results show that only twofold degeneracy is found in all states.
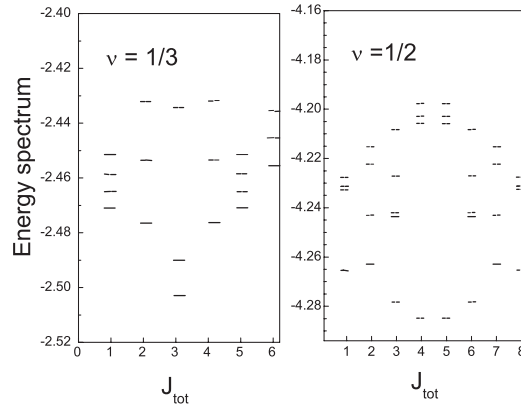
Figure 1. Low-lying energy spectra in the lowest LL for $\nu = 1/2$ with nine electrons and $\nu = 1/3$ with six electrons, respectively. The shorter dashes indicate degenerate states with the same J_{tot}. The number of dashes gives the degeneracy.

3. Pair-correlation functions

The pair-distribution function of the electrons at the topmost LL n is given by

$$g_n(\vec{r}) = \frac{ab}{N_n(N_n-1)} \langle \Psi_n| \sum_{i \neq j} \delta(\vec{r} + \vec{r}_i - \vec{r}_j)|\Psi_n\rangle = \frac{1}{N_n(N_n-1)}$$

$$\times \sum_{\vec{q},\{j\}} e^{[i\vec{q}\cdot\vec{r} - \vec{q}^2 l^2/2 - iq_x(X_{j_1} - X_{j_3})]} \delta'_{j_1+j_2,\,j_3+j_4} \delta'_{j_1-j_4,\,q_y b/2\pi} \langle \Psi_n| a_{j_1}^\dagger a_{j_2}^\dagger a_{j_3} a_{j_4}|\Psi_n\rangle, \quad (7)$$

while the wavefunction $|\Psi_n\rangle$ is written by a linear combination of the bases as

$$\Psi_n = \sum_{\{j\}} c(j_1, j_2, \ldots, j_{N_n}) a_{j_1}^\dagger a_{j_2}^\dagger \cdots a_{j_{N_n}}^\dagger |0\rangle. \quad (8)$$
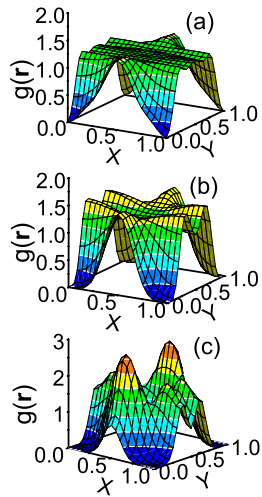
3.1. Correlations in the lowest LL

Figure 2 displays the ground-state pair-correlation functions in the lowest LL. For $\nu = 1/2$ and 1/3 the correlation functions show no spatial crystalline symmetry. However, we note that there is a remarkable distinction between these two liquid-like states. In the $\nu = 1/2$ state, electrons can approach each other with high probability; this is a common Fermi liquid feature. It is currently considered as a metallic CF sea state. On the other hand, in the $\nu = 1/3$ state the probability for electrons to approach each other is strongly quenched. This is the typical characteristic of the famous Laughlin liquid, which is described by a variational wavefunction as

$$\Psi(\vec{r}_1, \vec{r}_2, \ldots) = \prod_{j \neq k} (z_j - z_k)^3 \exp\left[-\sum_j r_j^2/4l^2 \right], \quad (9)$$

where $z_j = x_j - y_j$ is the complex coordinate of the ith particle. Because of the shape of our sample, we cannot directly compare the overlap of the ground-state wavefunction between our result and equation (9).

As shown in figure 2(c), the $\nu = 1/7$ state exhibits a hexagonal structure, indicating that a WC is formed. We note the oscillation peaks are not uniform, implying that the crystalline state

Figure 2. The ground-state pair-correlation functions in the lowest LL for (a) $\nu = 1/2$, (b) $\nu = 1/3$, and (c) $\nu = 1/7$, respectively. The coordinates are normalized by the dimension of the cell: $(X, Y) = (x/a, y/b)$.
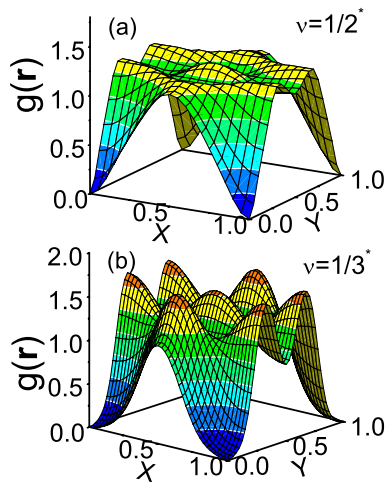
Figure 3. Same as in figure 2 for one of the low-lying excited states at $\nu = 1/2$ and $1/3$. The crystal characteristic can be clearly identified.

may not be the naive WC but the more general CDW. In the latter configuration, there may be several electrons that reside in a lattice site (form a 'bubble'). In our calculation, the $\nu = 1/7$ state seems to reveal the feature of a superlattice. We are not certain whether this result is a novel CDW state or just a finite size effect.

Figure 3 reveals that for some low-lying excited state, the pair-correlation functions also reveal a crystalline structure at $\nu = 1/2$ and $1/3$ as well. The crystalline characteristic at $\nu = 1/2$ is less obvious than at $\nu = 1/3$, because the overlap between wavefunctions at adjacent lattice sites are larger. The broken symmetries in excited states reflect that the magneto-rotons have wavevectors similar to that of the WC.
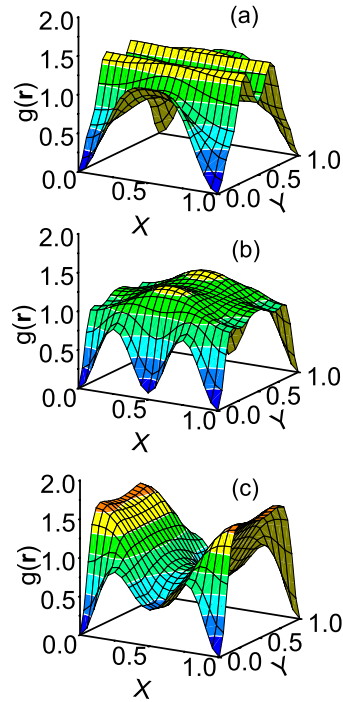
Figure 4. The $\nu_n = 1/2$ pair-correlation functions for higher lowest Landau levels: (a) $n = 1$; (b) $n = 2$; (c) $n = 3$.

3.2. Correlations in higher LLs

In figure 4, we show the pair-correlation functions in higher LLs at $\nu_n = 1/2$ or $\nu = 2n + 1/2$. We calculate nine electrons in the topmost LL. For $n = 1$, there is no indication of the crystalline symmetry. The state is nearly liquid-like, which coincides with experimental observations. Theoretically, the $\nu = 2 + 1/2$ state is explained as a paired composite Pfaffain state, which shows a quantum Hall plateau at the even-denominator filling factor accompanying a energy gap. For $n = 2$, the crystalline feature appears. For even higher LLs ($n \geqslant 3$), the pair-correlation function reveals a unidirectional translation symmetry, which is the familiar unidirectional CDW or stripe phase at the half-fillings in high LLs. These states were first predicted by Koulakov *et al* [22] in the limit of large n, using Hartree–Fock mean-field theory, and they were recently observed in a series of experiments [20, 21]. The experimental anisotropic phenomena occur at $n \geqslant 2$, while ours are at $n \geqslant 3$ (figure 4(c)). The difference may result from the finite number of electrons employed in our computations. Nevertheless, the numerical results coincide qualitatively with the experimental observations.

Figure 5 shows that the pair-correlation functions in higher LLs at $\nu_n = 1/3$ or $\nu = 2n + 1/3$. Accordingly, the $\nu = 2 + 1/3$ state exhibits no signs of crystalline symmetry. At $n = 2$ (figure 5(b)), we find that the correlation function reveals the characteristic of the so-called bubble phase. This phase has been studied extensively in the literature [22, 29]. The stripes at $\nu_n = 1/3$ are formed at higher LLs than at $\nu_n = 1/2$, because the electron density is smaller. In figure 5(c), stripes are found at $n \geqslant 4$, which lead to the anisotropic magneto-transport in 2D quantum Hall systems.
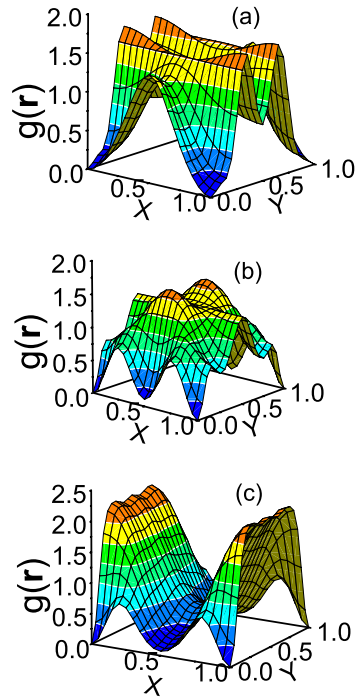
Figure 5. Same as in figure 4 for $\nu_n = 1/3$: (a) $n = 1$; (b) $n = 2$; (c) $n = 4$.

4. Discussion

We have systemically calculated the pair-correlation functions for a finite electron system on a rectangular sample. We recovered the rich phase structure for 2D electrons in the quantum Hall regime in the lowest LL as well as in higher LLs. Our results qualitatively coincide with the current understanding of relevant phenomena and experimental observations. In the lowest LL, we find that hexagonal WCs exist in the low-lying excited states at $\nu = 1/2$ and $1/3$ as well. For higher LLs, we only consider correlations between electrons in the topmost LL whereas we omit correlations between electrons in different LLs. The screening effects caused by electrons in the lower filled LLs are neglected. For higher LLs, the Laughlin liquids eventually disappear and are replaced by CDWs. In particular, the stripe phase dominates over other phases for $n \gg 1$. The correlation functions also coincide with the variational Monte Carlo results carried out in [29]. Some marginal distinctions mainly originate from the finite size effects of the exact diagonalization method.

Acknowledgment

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